

## **Estimating the Economic “Trade” Value of Increased Transmission Capability**

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## **I. Introduction**

Electricity markets are different in a number of ways from other types of markets. One crucial difference is that, unlike many other products, the ability to trade electricity between different geographic areas is physically constrained by “transportation” (*i.e.*, transmission) limits. Another difference is that estimating the value of increased inter-regional transmission capability is difficult, because of complications in determining when physical or institutional barriers to electricity trading are actually encountered.

Regulators have long been concerned about the lack of market incentives for providing efficient levels of electricity transmission capacity. Due to its “natural monopoly” characteristics, where a single provider of transmission within a specified geographic area enjoys scale and scope economies and other network efficiencies (*e.g.*, internalization of loop flows and increased reliability arising from synchronized control of transmission resources), transmission and distribution companies covering a specified area traditionally have been regulated to ensure adequate investment and system reliability. Nonetheless, concerns abound that the current U.S. transmission system is inadequate to handle the demands created by a deregulated wholesale electricity trading environment.<sup>1</sup>

Despite widespread concerns about transmission adequacy, little empirical work has attempted to estimate the value of additional transmission capability. Here, we describe an innovative methodology that uses market data to determine the economic value of increased transmission capability between two geographic areas. Our analysis focuses specifically on electricity trade between the New York ISO and ISO–New England.

## **II. Potential Market Limitations in Providing Adequate Electricity Transmission Capacity**

Transmission lines are the vehicle that permits the exportation and importation of electric power from one geographic area to another. Transmission lines are potentially valuable when the “natural” or “autarky” price of electricity differs across regions. This might occur, for example, when a region with peak electricity demand in the summer is located near another region with peak demand in the winter. More generally, the vintage, technology, and fuel source of electricity plants differs across regions, which induces inter-regional differences in supply conditions. In this situation, transmission capacity allows lower-cost electric generation to be exported into areas that would otherwise be served by higher-cost local generation during many hours of the day and year. Thus, transmission capacity allows for some degree of arbitrage in supplying power to energy-consuming regions, leading to potentially increased productive efficiency as lower-cost generation is used to replace higher-cost generation.

Unfortunately, the provision of electricity transmission (and distribution) services is vulnerable to market failure. In particular, the economies of scale and scope involved in constructing an electric grid to efficiently handle geographically dispersed electricity demand and supply sources arguably make electricity transmission services conducive to natural monopoly. Moreover, when facing unregulated pricing of transmission services, the owners of transmission networks would not have economic incentives to efficiently mitigate transmission congestion. That is because transmission lines need to be congested so that sufficient revenues are earned under unregulated pricing to cover the cost of building and maintaining transmission facilities.

Assume, for example, that a transmission line exists for transporting power between Region 1 and Region 2 which is capable of handling energy flows up to 100 MW. The willingness of energy traders to pay for use of this transmission capacity is equal to the price difference between the two regions, less direct trading costs. If, however, sufficient transmission capacity is available, then trade will cause prices (net of trading costs) to equalize across the two regions. Since there would be no profitable marginal

trading opportunity in this case, traders would be unwilling to pay for transmission capacity, and the transmission owner would be unable to gain revenues. Thus, in an unregulated environment, a transmission owner typically would capture revenue only when transmission usage is constrained, which allows the price in Region 1 to rise above the price in Region 2 (or vice versa). In this paper, we refer to this state as “constrained trading.”

Since the presence of network economies makes it difficult for the market to sustain profitable entry, doubts exist that competition would provide efficient levels of transmission capacity. As a result, owners of transmission and distribution facilities are typically regulated in the United States (and most other countries), thereby placing the burden on federal and state authorities to determine appropriate transmission investment.

Much ink has been spilt on how to address transmission adequacy in the current U.S. environment, where transmission owners are regulated but wholesale electricity trading is largely deregulated and placing ever increasing demands on the transmission infrastructure. We suggest, however, that in order to have a proper debate as to what type of transmission expansion might be economically warranted, a method must first be devised to estimate the economic value of transmission capacity.

### **III. The Impact of Electricity Trade on Market Prices in the Presence of Transmission Limits**

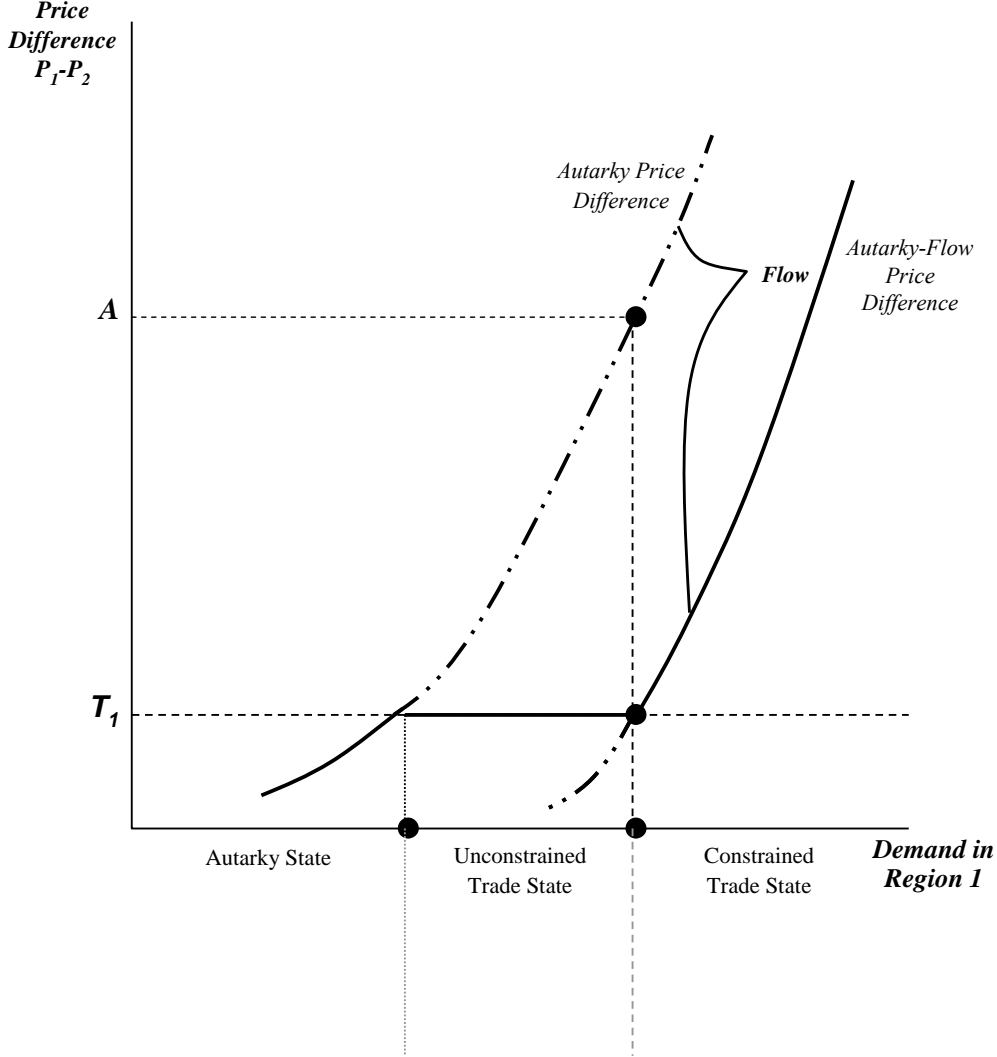
To determine whether transmission capacity is scarce at a particular time, one must determine whether energy trading appears to be volume-constrained. Merely looking at energy flow data is not necessarily sufficient to make this determination. Since thermal limits constrain the amount of electricity that can flow across a particular transmission line, available transmission capacity varies with temperature conditions. In addition, other issues, such as dynamic flow instability, may constrain use of the transmission system.

Moreover, quantity constraints on electricity flows could arise not only from reaching physical transmission limits, but also from institutional obstacles, such as coordination, control, and informational impediments that keep available transmission capacity from being used effectively. For these reasons, we use market pricing data to measure transmission value and to identify inter-regional price differences consistent with binding quantitative constraints on energy trading.

Given a specific inter-regional price difference (*e.g.*, between ISO-New England and the New York ISO) observed at particular time, it must hold logically that this difference represents one of three possible “states”: (1) no short-term inter-regional trade (*i.e.*, autarky); (2) unconstrained trade (*i.e.*, arbitrage); and (3) quantity-constrained trade (*i.e.*, transmission constrained trade). The autarky state reflects the absence of significant short-term trade (although trade may occur under longer-term contract arrangements), while the arbitrage state is consistent with trading without a binding constraint on the volume traded. Finally, quantity-constrained trade represents a state where trade takes place up to some capacity constraint, such as transmission limits. Our methodology looks at electricity price differences, weather conditions, and fuel prices to determine which of these states is most likely being observed on a particular day.

Figure 1 illustrates the evolution of the three equilibrium states as demand increases in Region 1, holding demand conditions in Region 2 and supply conditions in both regions constant. For expositional convenience, the figure illustrates only those demand conditions where Region 1’s price exceeds Region 2’s price in the absence of trade. The solid line shows the equilibrium price difference between the two regions, as well as which of the three equilibrium states (*i.e.*, autarky, unconstrained trade, and quantity-constrained trade) prevails. The determination of this equilibrium state depends on the magnitude of the price difference between the two regions in the absence of short-term trade, which is represented by the *Autarky Price Difference* line. It also depends on the trading cost  $T_t$ , and the effect on the inter-regional price difference if trade occurs up to the transmission system’s limits (as described by the variable, *Flow*).

Figure 1  
 Three Possible Equilibrium States  
 (Holding Demand in Region 2 Constant)



When Region 1's demand is at sufficiently low levels, the inter-regional price difference in the absence of short-term trade (*i.e.*, autarky) is less than the inter-regional trading cost,  $T_l$ . Hence, no short-term trade occurs between regions, implying that the observed price difference is consistent with autarky.

As demand grows in Region 1 (*e.g.*, as the temperature increases during the summer), we move rightward along the horizontal axis in Figure 1, leading to larger differences between Region 1's price and Region 2's price under autarky. When the autarky price difference equals the inter-regional trading cost  $T_l$ , trade begins and electricity is exported from Region 2 into Region 1.

With continued demand growth in Region 1, the state of unconstrained trade continues and the inter-regional price difference remains at  $T_l$  if trading does not fully exhaust the inter-regional transfer capability. In other words, if the inter-regional price difference under autarky exceeds the trading cost  $T_l$ , and if the inter-regional price difference is less than  $T_l$  if electricity trade occurred up to the transmission system's capacity limits, then unconstrained trading will arise and induce a price difference equal to  $T_l$ . This outcome is illustrated in Figure 1 for those demand levels where the *Autarky Price Difference* line is above  $T_l$  and the *Autarky-Flow Price Difference* (which represents the price difference when trade takes place up to the transmission system's limits) is below  $T_l$ .

As demand increases still further, the trade volume arising under unconstrained trade eventually equals the transmission system's capacity limits. At this point, the price difference  $T_l$  under unconstrained trade equals the price difference that would arise if trade was permitted up to the transmission system's limits (as denoted by the line, *Autarky-Flow Price Difference*). Further increases in demand in Region 1 from this level result in a constrained-trade state, where transmission is economically scarce. The observed interregional price difference, denoted by the *Autarky-Flow Price Difference* line in Figure 1, now exceeds the unconstrained trading cost,  $T_l$ .

The above exposition leaves us three things to estimate statistically if we are to measure the economic “trade” value of additional transmission capacity. We need to estimate an autarky price-difference equation, the unconstrained trading cost, and the amount that the inter-regional price difference is reduced (from autarky) if trade takes place up to the transfer capability limits. In our technical working paper on this subject,<sup>2</sup> we show how to estimate these relationships. We provide a simplified explanation of this technique in the next section.

#### **IV. A Simplified Technical Explanation: Determining Unconstrained Trading Costs and the Likelihood of Transmission–Constrained Trade**

As expressed below, we assume that the daily autarky price difference  $A_t$  between Region 1 and Region 2 (*i.e.*, Region 1’s price less Region 2’s price) is a function of that day’s temperature, fuel (*i.e.*, natural gas) prices on that day, and a “noise” term:

$$A_t = \beta_0 + \beta_1 \text{temperature}_t + \beta_2 \text{fuel price}_t + \varepsilon_t. \quad (1)$$

The  $\beta$ s represent coefficient values that are estimated statistically, and the term  $\varepsilon_t$  is a normally distributed noise term (whose standard deviation is also estimated statistically). The temperature variable, representing the high temperature for the day, is actually divided into several variables that represent heating and cooling degree days and their squared values.

In addition, as represented below, we assume that trading costs  $T_{1t}$  on a particular day equal a constant  $\theta_0$ , plus a noise term  $\eta_t$ :

$$T_{1t} = \theta_0 + \eta_t. \quad (2)$$



The term  $\eta_t$  is normally distributed, but truncated below at  $-\theta_0$ . This truncation implies that trading costs are always positively valued.

Under transmission-constrained (*i.e.*, quantity-constrained) trade, the inter-regional price difference equals the autarky price difference less the reduction in the inter-regional price difference that occurs when electricity flows up to the transfer capability limits. We denote this reduction by the variable, *Flow*, implying that the price difference  $C_t$  under transmission-constrained trade equals the following:

$$C_t = A_t - Flow. \quad (3)$$

Now, consider an observed price difference  $Y_t$ . This observed difference may represent an autarky outcome (*i.e.*,  $Y_t = A_t$ ), where trading costs necessarily exceed (or equal)  $Y_t$  (*i.e.*,  $Y_t \leq T_{lt}$ ). Alternatively, the observed price difference may represent unconstrained trade (*i.e.*,  $Y_t = T_{lt}$ ), where the autarky price difference necessarily exceeds the observed price difference and the price difference under transmission-constrained trade would be less than (or equal to) the observed price difference (*i.e.*,  $C_t \leq Y_t < A_t$ ). Finally, the observed price difference may represent transmission-constrained trade (*i.e.*,  $Y_t = C_t$ ), where trading costs are necessarily below the observed price difference ( $Y_t > T_{lt}$ ). Summarizing, we have three possible consistent outcomes:

<u>Autarky</u> :	$Y_t = A_t$ and $Y_t \leq T_{lt}$ .	
<u>Unconstrained Trade</u> :	$Y_t = T_{lt}$ and $C_t \leq Y_t < A_t$	(4)
<u>Constrained Trade</u> :	$Y_t = C_t$ and $Y_t > T_{lt}$ .	

The probability of being in a particular equilibrium state, such as autarky, constrained trade, or unconstrained trade, depends on the observed price difference, the temperature and fuel price levels at that particular time, the estimated coefficients  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ ,  $\theta_0$ , and *Flow*, and the estimated standard

deviations of the noise terms,  $\varepsilon_t$  and  $\eta_t$ . A statistical technique called “maximum-likelihood estimation” can be used to derive all of the coefficient estimates. These include estimates of the level of unconstrained trading costs between the two regions, which are allowed to differ based on the direction of trade (*i.e.*, based on which region has the higher price on a particular day).

Since we have assumed that  $\varepsilon_t$  and  $\eta_t$  are normally distributed, we can use the properties of the normal distribution, as well as the parameter estimates and variable levels, to determine the probability that transmission constraints are binding on a particular day. Consistent with expectations, our parameter estimates imply that the probability of observing transmission-constrained trade is higher when temperatures are at particularly high, or low, levels. For example, transmission-constrained trade between New York and New England arises typically in summer or winter months.

## **V. Data**

Our analysis focuses on inter-regional trading costs between ISO-New England and the New York ISO. The data used to perform our estimation includes three key components: (i) electricity prices, (ii) temperatures, and (iii) fuel costs. The electricity prices (\$ per MWh) are volume-weighted averages of the contract prices for pre-scheduled, day-ahead 16-hour blocks of electricity.<sup>3</sup> The data are for the period from March 1997 through June 2002, comprising 1350 observations. The ISO-New England price series is for power delivered anywhere within the ISO’s boundaries, while the price series relevant to the New York ISO is for power bought and sold in Western New York (Zone A, which excludes New York City and Long Island). Our price difference variable is the New England (1x16) price less the New York (1x16) price.

We use temperature as the key demand shifter for electric power. Electric power consumption increases substantially at high temperatures, particularly through increased demand for air conditioning

and other cooling processes. At particularly low temperatures, electricity demand for heating purposes increases as well. Since ISO-New England and New York ISO cover relatively broad geographic areas, a weighted-average temperature for each region was calculated using the daily maximum temperature in major cities,<sup>4</sup> where the weights equaled the population of the corresponding metropolitan area. Daily maximum temperatures were obtained from the National Climatic Data Center.<sup>5</sup>

We then take the weighted-average high temperature for a given day and convert it into cooling and heating degree days, where “cooling degree days” equals  $\max(0, \text{temperature}-65)$ , and “heating degree days” equals  $\max(0, 65-\text{temperature})$ . Our statistical analysis uses the difference in cooling (or heating) degree days between New England and New York as an explanatory value in determining the inter-regional price difference in the absence of trade. To allow for an increasing impact of temperature changes on inter-regional price differences under more extreme conditions, we also include the difference in the squared values of cooling (or heating) degree days.

Natural gas constitutes an important fuel source, and consequently an important short-run cost, for many power generators. We rely on the daily cash-market closing price at Henry Hub (a major trading point in Louisiana). Although coal is an important fuel source for some generators, it is typically purchased on a contract basis, with substantial regional differences in prices. Since there is no day-ahead spot market for coal transactions, we assume that the coal price on a given day is related to the gas price on that day. Certainly, for an operator of a coal-fired power plant, its willingness to pay for coal on a given day is influenced by the price of electricity on that day, and the latter is influenced by the price of natural gas facing its competitors.<sup>6</sup>

## **VI. Estimation Results: Unconstrained Trading Costs and the Probability of Encountering Transmission Constraints**

To save space, we will not present specific coefficient estimates resulting from our estimation of the model described above. However, the results from our statistical model indicate that the price difference between New England and New York increases substantially along with the price of natural gas in the absence of trade. This finding suggests that, on the margin, New England is more dependent than New York on natural gas as a fuel source for supplying electricity. Also, our estimation shows that as summer temperatures in New York rise substantially above those in New England, electricity prices increase in New York relative to New England in the absence of unconstrained trade (and vice versa). In addition, as winter temperatures fall in New England substantially below those of New York, electricity prices rise in New England relative to New York.

However, we are fundamentally more interested in estimating unconstrained trading costs between New York and New England and the value of adding more transmission capability between the two regions. Accordingly, Table 1 presents estimates of the unconstrained energy trading costs involving these regions. These costs represent not only transmission charges but other unobserved costs involved in electricity trading, such as the information cost involved in identifying a willing buyer or seller and any “opportunity” cost involved in making the electricity available to that buyer or seller.

**Table 1: Transaction Cost Estimates – New England /New York  
(\$ per MWh)**

<b>Time Period</b>	<b><u>Transaction Cost to New England from New York</u></b>	<b><u>Transaction Cost to New York from New England</u></b>
<b>March 1, 1997 – April 30, 1999</b>	<b>5.47</b>	<b>2.17</b>
<b>May 1, 1999 - November 17, 1999</b>	<b>4.90</b>	<b>1.24</b>
<b>November 18, 1999 – June 30, 2002</b>	<b>7.77</b>	<b>2.37</b>

We find that trading costs to New England from New York are substantially higher than trading costs in the opposite direction. The magnitude of the trading costs suggests that prices may need to be at least \$5-8 per MWh higher in New England to attract substantial short-term power exports from New York.

Curiously, trading costs between New England and New York declined modestly after ISO-New England began operation of its energy “exchange” market on May 1, 1999, although this decrease was not statistically significant. Perhaps, the formation of a more transparent market in New England facilitated price discovery and lowered trading costs between New England and New York. Conversely, the subsequent opening of the New York ISO’s energy exchange on November 18, 1999 is associated with a substantial and statistically significant increase in the cost of trading energy to New England from New York. This result is consistent with other research that we have performed, which indicates that ISO formation facilitates trade within the ISO’s boundaries, but may impede exports of energy to other areas.<sup>7</sup> This behavior can be brought about by ISO protocols that inhibit exports at particular times, or the institution of ISO-based capacity markets that inhibit generation resources participating in that market from sending power outside of the ISO.

Table 2 describes the probability of encountering binding trade volume constraints, such as transmission capacity limitations. To derive these results, we take the New England – New York price differential on a given day, along with the weather conditions and fuel prices for that particular day, and then use our coefficient estimates to determine the probability that the observed price difference could result from a state of quantity-constrained trade, unconstrained trade, or no trading.<sup>8</sup> For those days where the price in New England is higher than the corresponding price in New York, quantity-constrained trade arises approximately 11.5 percent of the time. For those days where the price in New York is relatively higher than New England, quantity-constrained trade arises approximately 25.3 percent of the time. It is much more common for prices to be higher in New England; the day-ahead 1x16 prices in New England exceeded those in New York on 87 percent of the days in our sample.

These results from Table 2 are consistent with the notion that transmission constraints are encountered somewhat frequently in sending power to New England from New York, due to the limited capacity of the interties connecting the regions. Congestion appears to be an even greater issue on those infrequent days when power is sent to New York from New England, since the probability of encountering binding trade volume constraints is even greater.

**Table 2: Estimated Average State Probabilities –New England/New York**

<b>Regime</b>	<b><u>When New England Price Is Higher Than New York Price</u></b>	<b><u>When New York Price Is Higher Than New England Price</u></b>
<b>Autarky</b>	<b>5.5%</b>	<b>9.3%</b>
<b>Unconstrained Trade</b>	<b>83.0%</b>	<b>65.4%</b>
<b>Quantity-Constrained Trade</b>	<b>11.5%</b>	<b>25.3%</b>

## **VII. Estimating the Economic “Trade” Value of Increased Transmission Capacity**

Our calculation of the economic value of increased transmission capability is straightforward. At the time when transmission constraints are binding, the marginal value of additional transmission capability between two regions is merely the difference in prices between the two regions, less the estimated unconstrained trading cost. We use this measure because, if a state of quantity-constrained trade is being observed, an incremental increase in electricity flows from a lower-priced region to a higher-priced region will reduce energy procurement costs by the observed price difference, less the applicable transaction cost. However, since we cannot determine exactly when transmission constraints are binding, this estimate must be multiplied by the estimated probability that we are observing a transmission-constrained trading state.

Therefore, a two-step approach is used to estimate the marginal economic value of increased transmission capability between New York and New England. First, we take the observed inter-regional price difference on each day and subtract our estimated mean transaction cost. Second, we take this daily estimate of the marginal value of increased transmission capability (contingent on transmission constraints that are binding on that day), and multiply it by our estimated probability that the observed inter-regional price difference for that day is associated with transmission-constrained trade. Of course, the probability of observing transmission-constrained trade depends on the inter-regional price difference, temperature conditions, fuel prices, and other factors for that particular day.

**Figure 2: The Economic "Trade" Value of Additional Transmission Capacity  
(Monthly Value in \$/MW from March 1997 - June 2002)**

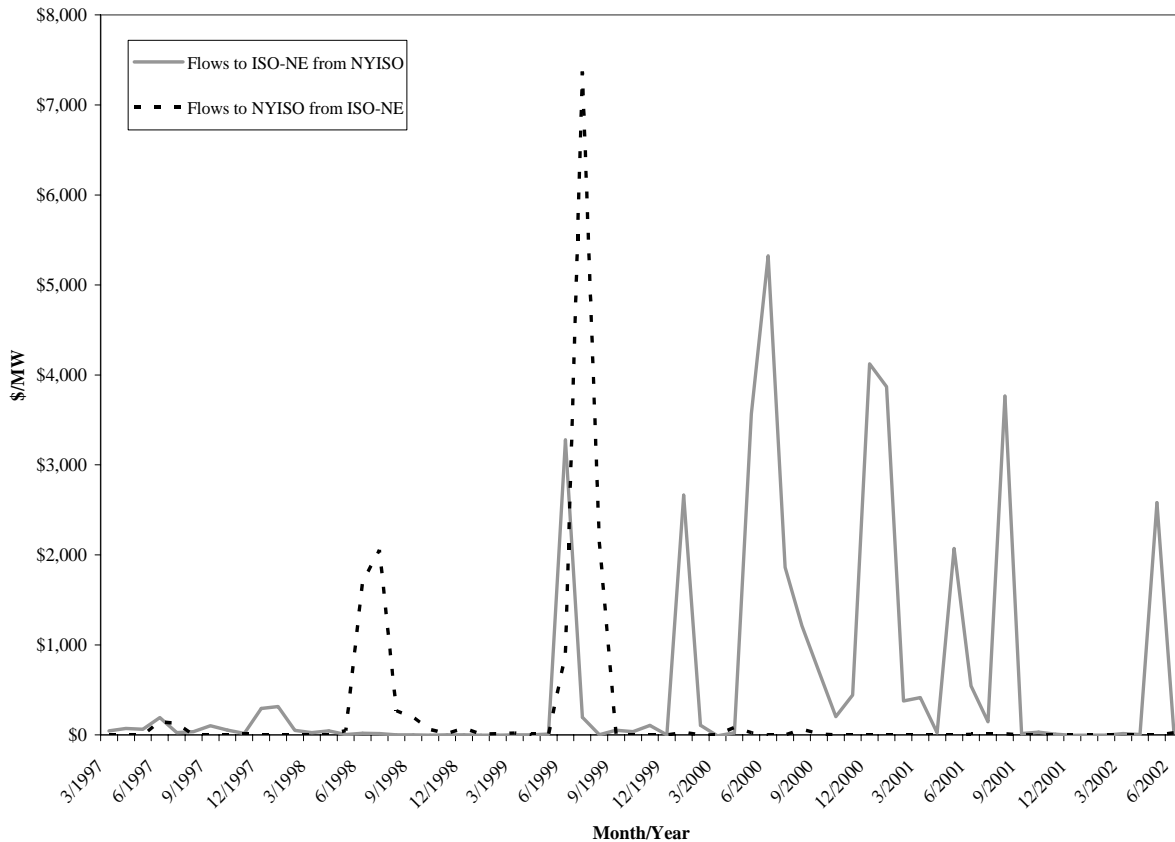


Figure 2 shows the aggregation of these daily estimates into a monthly estimate of the economic value of adding an additional megawatt of transfer capability between New York and New England. Unlike electricity trade in many other regions of the United States, where additional transmission capacity can provide substantial value by mitigating summer price “spikes” in areas affected by high load conditions, the addition of transmission capacity between New York and New England also provides significant economic value in winter months. Given that electricity prices in New England were consistently and substantially higher than New York in January and December of 2000, as well as in January 2001, additional transfer capability would have been valuable in those months. Also, it is interesting to note that additional transfer capability for sending power to New York from New England would have provided substantial value during the summers of 1998 and 1999, while most of the value of



additional transfer capability during the summers of 2000 and 2001 would have been for sending power in the opposite direction (*i.e.*, to New England from New York).

As shown in Table 3, our estimates indicate that for the five and one-third years contained in our data sample, the annual average economic “trade” value of additional transmission capability was \$7,341 per MW/year for sending power to New England from New York. During the same period, this value was \$2,892 per MW/year for sending power to New York from New England. This result suggests that New England energy consumers are likely to benefit more from increased transmission capacity than New York energy consumers. As shown previously (see Figure 2), these annual values depend largely on temperature conditions in the winter and summer months, which are quite variable from year to year. Thus, to properly estimate the economic value of additional transmission capacity, the period under review must be sufficiently extensive to capture the impact of particularly high load conditions on market behavior.

**Table 3: Annual Economic “Trade” Value of Additional Transmission Capacity  
(Average \$ per MW/year: March 1997 – June 2002)**

<b>To:</b>	<b>ISO - New England</b>	<b>New York ISO</b>
<b>From:</b>	<b>New York ISO</b>	<b>ISO - New England</b>
	<b>\$7,341</b>	<b>\$2,892</b>

## **VIII. Conclusion**

We have devised a model with parsimonious data requirements that assesses whether observed inter-regional differences in electricity prices are associated with autarky (*i.e.*, no short-term trade), arbitrage (*i.e.*, unconstrained trade), or a transmission-constrained (*i.e.*, quantity-constrained) trading equilibrium. By expanding prior trading-cost models to allow for the possibility of a binding restriction on trade

volume, our methodology allows us to form estimates of the economic “trade” value of increased electricity transmission capacity. That is, we estimate the potential gains that can be achieved through transmission expansion in moving power from lower-priced to higher-priced regions. Rather than relying on simulation approaches, where electricity is dispatched based on specific assumptions regarding market bidding behavior, plant generating costs, and plant availability, our analysis instead relies on actual market data to estimate this source of economic value.

Our estimates do not include the “reliability” value of increased transmission capacity, which considers how additional transmission capacity may reduce the probability of system outages that induce economic losses as a result of lost load. Although economic estimates of reliability impacts have received significant past attention, more research in this area is nonetheless warranted.

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<sup>1</sup> See, among others, U.S. Department of Energy, “National Transmission Grid Study,” May, 2002.

<sup>2</sup> See Andrew N. Kleit and James D. Reitzes, “FERC’s Transmission Policy and Transmission Adequacy: When Is Transmission Scarce?” manuscript, July 2005. An earlier version of this manuscript, entitled “Geographic Integration, Transmission Constrains, and Electricity Restructuring,” is available from the Social Science Research Network ([www.ssrn.com](http://www.ssrn.com)), Manuscript No. 645085. That paper was presented at the 10<sup>th</sup> Annual POWER Research Conference on Electricity Industry Restructuring held by the University of California Energy Institute.

<sup>3</sup> *Power Markets Week* collects and compiles transaction price information as reported by energy traders.

<sup>4</sup> With the New York ISO, we used temperatures for Buffalo and New York City. With ISO–New England, we used temperatures for Providence, RI; Hartford, CT; Boston, MA; Concord, NH; Portland, ME; and Burlington, VT.

<sup>5</sup> For further description, see <http://lwf.ncdc.noaa.gov/oa/ncdc.html>.

<sup>6</sup> This potentially raises the issue that the regional price of coal and natural gas may be endogenously determined, which raises issues of possible bias in the statistical estimation. To mitigate this potential problem, we use prices for natural gas delivered to Henry Hub (Louisiana), a major trading point. Prices at Henry Hub are not likely to be substantially influenced by conditions affecting electricity markets in the eastern United States.

<sup>7</sup> See Andrew N. Kleit and James D. Reitzes, “FERC’s Transmission Policy and Transmission Adequacy: When Is Transmission Scarce,” manuscript, July 2005.

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<sup>8</sup> The probability of observing constrained trade is merely the likelihood of observing that specific price difference in the constrained trade state, divided by the sum of the likelihood of observing that particular price difference in all three states (for those particular temperature and fuel price conditions). This is an application of Bayesian statistics.